Problem Set 2 for Econometrics

1 Let

$$x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \text{and} \quad y = \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

Find the following,

(a) the orthogonal projection on range(x).

(b) the orthogonal projection of y on range(x).

(c) (optional) the projection on range(x) along the direction of y.

2 Consider the simple linear regression, $y_i = \beta_0 + \beta_1 x_i + u_i$. Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the OLS estimators, and denote the sample average of y_i by $\bar{y}$. Define $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$, $\hat{u}_i = y_i - \hat{y}_i$, and

$$\begin{aligned} T &= \sum_{i=1}^{n} (y_i - \bar{y})^2 \\ E &= \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 \\ R &= \sum_{i=1}^{n} \hat{u}_i^2 \end{aligned}$$

Prove the following identity:

$$T = E + R.$$

3 Solve the following minimization problem,

$$\min_{\beta} (Y - X\beta)' \Omega (Y - X\beta),$$

where Ω is a symmetric positive definite matrix. Note that this problem reduces to OLS if $\Omega = I$.

4 Consider a linear regression $y_i = x_i'\beta + u_i$. Some elements of x_i are, however correlated with u_i. Now we have another vector of variables z_i that satisfy $\mathbb{E} z_i u_i = 0$ and $\mathbb{E} z_i x_i'$ is invertible. Derive a method of moment estimator for β.

5 Assume that $x_1, \ldots, x_n$ are i.i.d. Uniform($[0, \theta]$). That is, the density function of each x is given by

$$p(x|\theta) = \begin{cases} \frac{1}{\theta} & 0 \le x \le \theta \\ 0 & \text{otherwise} \end{cases}$$

Derive the maximum likelihood estimator for the parameter θ.