Econometrics Lab 3

Monte Carlo Simulations

1. Heteroskedasticity-Robust Standard Error (HRSE) In this exercise, we use Monte Carlo simulations to show that the HRSE makes the distribution of the Student t statistic closer to N(0,1). Consider the following model,

$$y_i = \beta_0 + \beta_1 x_i + u_i, \quad i = 1, \ldots, n \tag{1}$$

where $u_i = \sqrt{0.5 + x_i^2} \cdot \varepsilon_i$, ε_i is i.i.d. N(0,1), and n is sample size. Let $\beta_0 = \beta_1 = 1$.

(1) Let n = 10, 100, and 1000. In each case, generate data (Y, X) and estimate the slope using OLS repeatedly for 5000 times, and calculate the mean and standard deviation of the estimates. And draw histograms of the slope estimates.

(2) Now consider the following test:

$$\mathrm{H}_0 : \ \beta_1 = 1 \quad \mathrm{H}_1 : \ \beta_1 \neq 1.$$

Use simulations to show that the usual t statistic ($t_1 = \frac{\hat{\beta}_1 - 1}{se(\hat{\beta}_1)}$) deviates from N(0,1), even when the sample is large (ie, $n = 1000$).

(3) Now we use the HRSE to construct a new t statistic, $t_2 = \frac{\hat{\beta}_1 - 1}{hrse(\hat{\beta}_1)}$. Use simulations to show that t_2 is closer to N(0,1), especially when the sample is large.

(4) Use simulations to evaluate the type I and II errors of the tests based on t_1 and t_2. When evaluating the type II error, let the true value of β_1 be 1.5.