

## Math Problems

EC 311  
Junhui Qian

1. Consider the following system of equations,

$$\begin{aligned}x_1 + 0.5x_2 &= b_1 \\ -0.5x_1 + x_2 + 1.5x_3 &= b_2 \\ x_3 &= b_3,\end{aligned}$$

where  $(x_i)$  are unknowns and  $(b_i)$  are constants.

- (i) Solve for  $x_1$ ,  $x_2$ , and  $x_3$  using the Cramer's rule (google it).
- (ii) Represent the system of equations in matrix form,  $Ax = b$ , where  $A$  is a 3-by-3 matrix,  $x = (x_1, x_2, x_3)'$ , and  $b = (b_1, b_2, b_3)'$ .
- (iii) Invert  $A$  using a computer (you can use Mathematica, Matlab, Maxima (free), etc.), check whether your result is consistent with what you get in (i).

- 2 Consider the following function,

$$f(x, y) = 6x^{2/3}y^{1/2}.$$

Suppose that  $x = 998$  and  $y = 101.5$ , estimate  $f$  without using calculator. (hint: use total differentiation.)

- 3 Consider the following nonlinear equation,

$$x^2 - 3xy + y^3 = 7.$$

- (i) Check that the point  $(4, 3)$  is on the curve defined by the equation.
- (ii) Calculate the slope of the curve at  $(4, 3)$ . (hint: use the implicit function theorem)
- (iii) Suppose  $x$  increases to  $x = 4.1$ , estimate how much change occurs to  $y$  accordingly. And compare your estimate with true value that you can obtain using a computer.

- 4 Consider the following system of nonlinear equations,

$$\begin{aligned}y &= f(x, y) + g(y) + z \\ m(x, y) &= m_0,\end{aligned}$$

where  $f$ ,  $g$ , and  $m$  are all continuously differentiable. We regard  $x$  and  $y$  as endogenous (unknown) variables, and  $z$  and  $m_0$  as exogenous (given) variables.

- (i) Suppose there is a shock to  $m_0$ , say  $\Delta_m$ , estimate how much  $x$  is affected.
- (ii) Suppose there is a shock to  $z$ , say  $\Delta_z$ , estimate how much  $y$  is affected.