Problem Set 3 for Econometrics

due on the next lecture

EC 310
Junhui Qian

1 Let the matrix representation of the multiple linear regression be $Y = X\beta + U$. Suppose the covariance matrix of U is Ω. We may estimate β by solving

$$\min_{\beta}(Y - X\beta)'\Omega^{-1}(Y - X\beta).$$

The obtained estimator, $\hat{\beta}_{GLS}$, is called the GLS (Generalized Least Square) estimator.

(a) Prove that $\hat{\beta}_{GLS}$ is unbiased.

(b) Derive the covariance matrix of $\hat{\beta}_{GLS}$.

(c) (optional) Prove that $\hat{\beta}_{GLS}$ is BLUE.

2 (Problem 3.2 in Woodridge) Suppose we estimated the following model,

$$\widehat{edu} = 10.36 - 0.094\ sibs + 0.131\ medu + 0.210\ fedu. \quad n = 722, R^2 = 0.214,$$

where *edu* is years of schooling, *sibs* is number of siblings, *medu* is mother's years of schooling, and $fedu$ is father's years of schooling.

(a) Does *sibs* have the expected effect? Explain. Holding *medu* and $fedu$ fixed, by how much does *sibs* have to increase t o reduce predicted years of education by one year? (A noninteger answer is acceptable here.)

(b) Discuss the interpretation of the coefficient on *medu*.

(c) Suppose that Man A has no siblings, and his mother and father each have 12 years of education. Man B has no siblings, and his mother and father each have 16 years of education. What is the predicted difference in years of education between B and A.

3 Suppose that average worker productivity at manufacturing firms (*avgprod*) depends on two factors, average hours of training (*avgtrain*) and average worker ability (*avgabil*):

$$avgprod = \beta_0 + \beta_1 avgtrain + \beta_2 avgabil + u.$$

And suppose that CLR Assumptions 1-4 hold and that the government, in a drive for improving labor productivity, gives grants to those firms whose workers have less than average ability.

(a) Under the government program, is *avgtrain* negatively, or positively correlated with *avgabil*?

(b) If we estimate a simple regression of *avgprod* on *avgtrain*, and obtain $\tilde{\beta}_1$. Is $\tilde{\beta}_1$ biased upwards or downwards?