Stochastic Frontiers with Bounded Inefficiency[1]

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Abstract

This paper introduces a new model of stochastic production frontier that incorporates an unobservable bound for inefficiency, which is naturally instituted by the market competition. Our technical innovation is that we use doubly truncated normal, truncated half normal, and truncated exponential distributions to model the inefficiency component of the error term. We derive the form of density function for the error term of each specification and the formulas for calculating the conditional mean of the inefficiency levels. We also examine skewness properties of our new estimators which point to an explanation for the finding of "incorrect"skewness in many applied studies using the traditional stochastic frontier. We extend the model to the panel data setting and specify a time-varying inefficiency bound as well as time-varying efficiencies. A Monte Carlo study is conducted to study the finite sample performance of the maximum likelihood estimators in cross-sectional settings. We apply the model to a study of US banks from 1984 to 1995 utilizing a set of competing specifications of the stochastic frontier model.

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1 Introduction

The parametric approach to estimate stochastic production frontiers was introduced by Aigner, Lovell, and Schmidt (1977), Meeusen and van den Broeck (1977), and Battese and Cora (1977). These approaches specified a parametric production function and a two-component error term. One component, reflecting the influence of many unaccountable factors on production as well as measurement error, is considered "noise"and is usually assumed to be normal. The other component describes inefficiency and is assumed to have a one-sided distribution, of which the conventional candidates include the half normal (Aigner, et al., 1977), truncated normal (Stevenson, 1980), exponential (Meeusen and van den Broeck, 1977) and gamma (Greene 1980a,b, Stevenson, 1980).

All these formulations, however, allow unbounded support for the distribution of productive (cost) inefficiency term in the right (left) tail. This assumption is plausible if we study production units in a highly regulated or highly segmented market. It becomes unrealistic if we study competitive markets where the most inefficient units cannot exist for long. As long as there is some degree of competition and exit of highly inefficient firms from the market, production units constitute a truncated sample. The consequence is that even if we correctly specify a family of distributions for the inefficiency term, the stochastic frontier model may still be misspecified.

This paper introduces a new model of production frontiers that incorporates an unobservable upper bound for inefficiency. Technically, we introduce a truncation on the right tail of the distribution of the inefficiency component. We consider the doubly truncated normal, which includes the truncated half normal as a special case, and the truncated exponential distributions, at the cost of an additional parameter–the truncation threshold. The inefficiency upper bound, or the efficiency lower bound, can be consistently estimated by the maximum likelihood estimation along with other model parameters. This bound can naturally be used for gauging the tolerance for or ruthlessness against the inefficient firms. It is also worth mentioning that, using this bound as the "inefficient frontier," we may define "inverted" efficiency scores in the same spirit of "Inverted DEA" described in Entani, Maeda, and Tanaka (2002). We examine skewness properties of our new estimators. Our analysis points to an explanation for the finding of "incorrect"skewness in many applied studies using the traditional stochastic frontier and the potential for our bounded inefficiency model to explain these "incorrect"skewness findings. Researchers have often found positive instead of negative skewness in many samples examined in applied work, which may point to the stochastic frontier being incorrectly specified. However, we conjecture that the reason is that skewness is very hard to identify empirically when inefficiency is bounded. That is to say, when the true distribution of the one-sided inefficiency error is bounded (truncated) the extent to which skewness is present may be substantially reduced, often to the extent that negative sample skewness is not statistically significant. Thus the finding of positive skewness speaks to the weak identifiability of skewness properties in a bounded frontier model.

We also extend the model to the panel data setting and allow for time-varying inefficiency bound. By allowing the inefficiency bound to be time-varying, we in effect contribute another time-varying technical efficiency model. Our model differs from the existing literature in that, while previous time-varying efficiency models, notably Cornwell, Schmidt, and Sickles (1990), Kumbhakar (1990), Battese and Coelli (1992), and Lee and Schmidt (1993), are time-varying in the mean or intercept of individual effects, our model is time-varying in the lower support of the distribution of individual effects.

The outline of this paper is as follows. In Section 2 we present the new models and derive analytic formula for density functions and the calculation of inefficiencies. Section 3 deals with some technical issues of implementing parametric maximum likelihood for the bounded inefficiency model. In Section 4 we present Monte Carlo results on the finite sample performance of the bounded inefficiency model vis-a-vis classical stochastic frontier estimators. In Section 5 we give an illustrative study of the efficiency of US banking industry in 1984-1995. Section 6 concludes.

2 Models

We adopt the following Cobb-Douglas log-linear model,

$$y_i = A + \sum_{k=1}^{K} \alpha_k x_{i,k} + \varepsilon_i \tag{1}$$

where

$$\varepsilon_i = v_i - u_i. \tag{2}$$

For every production unit i, y_i is the log output, x_{ik} the k-th log input, v_i the noise component, and u_i the inefficiency component. We maintain the usual assumption that v_i is iid $N(0, \sigma_v^2)$, u_i is iid, and v_i and u_i are independent from each other and from regressors.

We subdivide the model in (1) into sub-models according to different specifications of u_i, the inefficiency component. If u_i is distributed as doubly truncated normal, we call the model *doubly truncated normal model,* and similarly for the *truncated half-normal model* and *truncated exponential model.* By the shape of probability density functions, there is an implicit assumption in the truncated half normal model and the truncated exponential model that most firms are relatively efficient. The doubly truncated normal model can be more flexible. It can describe the scenario that only a few firms in the sector are efficient, a phenomenon that is described in the business press as "few stars, most dogs". [2] In the following three subsections we provide details for the specifications of the three sub-models. In particular we derive the density functions for the error term ε_i, which is necessary for maximum likelihood estimation, and the analytic form for $E[u_i|\varepsilon_i]$, which is the best predictor of the inefficiency term u_i given knowledge of ε_i.

2.1 Doubly Truncated Normal Model

The first model we consider assumes that the inefficiency term has a doubly truncated normal distribution. Specifically, u_i has the following density function,

$$f_u(x) = \frac{\frac{1}{\sigma_u}\phi(\frac{x-\mu}{\sigma_u})}{\Phi(\frac{B-\mu}{\sigma_u}) - \Phi(\frac{-\mu}{\sigma_u})}\mathbf{1}_{[0,B]}(x),\ \ \sigma_u > 0, B > 0, \tag{3}$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are the cdf and pdf of the standard normal distribution respectively, and $\mathbf{1}[0,B]$ is an indicator function.

Note that the truncation parameter B can be a useful index of competitiveness of a market or an industry.

Next we derive the probability density function for the error term, ε, which can be used in maximum likelihood estimation. Since u and v are independent, the joint density function of u and v is

$$\begin{aligned} f_{u,v}(x,y) &= \frac{[\frac{1}{\sigma_u}\phi(\frac{x-\mu}{\sigma_u})][\frac{1}{\sigma_v}\phi(\frac{y}{\sigma_v})]}{\Phi(\frac{B-\mu}{\sigma_u}) - \Phi(\frac{-\mu}{\sigma_u})}\mathbf{1}_{[0,B]}(x) \\ &= \frac{\exp(-\frac{(x-\mu)^2}{2\sigma_u^2} - \frac{y^2}{2\sigma_v^2})\mathbf{1}_{[0,B]}(x)}{2\pi\sigma_u\sigma_v(\Phi(\frac{B-\mu}{\sigma_u}) - \Phi(\frac{-\mu}{\sigma_u}))}. \end{aligned} \tag{4}$$

The joint density function of u and ε is then

$$f_{u,\varepsilon}(x,y) = \frac{\exp(-\frac{(x-\mu)^2}{2\sigma_u^2} - \frac{(x+y)^2}{2\sigma_v^2})\mathbf{1}_{[0,B]}(x)}{2\pi\sigma_u\sigma_v(\Phi(\frac{B-\mu}{\sigma_u}) - \Phi(\frac{-\mu}{\sigma_u}))} \tag{5}$$

Integrate (5) with respect to x, we get the marginal distribution of ε,

$$\begin{aligned} f_\varepsilon(x) &= \left[\Phi(\frac{B-\mu}{\sigma_u}) - \Phi(\frac{-\mu}{\sigma_u})\right]^{-1} \cdot \left[\frac{1}{\sigma}\phi(\frac{x+\mu}{\sigma})\right] \cdot \\ & \left[\Phi(\frac{(B+x)\lambda + (B-\mu)\lambda^{-1}}{\sigma}) - \Phi(\frac{x\lambda - \mu\lambda^{-1}}{\sigma})\right], \end{aligned} \tag{6}$$

[2] We thank C. A. K. Lovell for providing us this link between our econometric methdology and the business press.

where

$$
\begin{aligned}
\sigma &= \sqrt{\sigma_u^2 + \sigma_v^2} \\
\lambda &= \sigma_u / \sigma_v.
\end{aligned} \tag{7}
$$

This is called λ-parameterization.

In practice we usually use another parameterization, called γ-parameterization,

$$
\begin{aligned}
\sigma &= \sqrt{\sigma_u^2 + \sigma_v^2} \\
\gamma &= \sigma_u^2 / \sigma^2.
\end{aligned} \tag{8}
$$

By definition $\gamma \in [0,1]$, this proves helpful in the numerical procedure of maximum likelihood estimation.

Note that when $B \to \infty$, (6) becomes

$$
f_\varepsilon(x) = \left[\Phi(\frac{\mu}{\sigma_u})\right]^{-1} \cdot \left[\frac{1}{\sigma}\phi(\frac{x+\mu}{\sigma})\right] \cdot \left[\Phi(\frac{\mu\lambda^{-1} - x\lambda}{\sigma})\right]. \tag{9}
$$

This is the probability density function for the Truncated Normal-Normal model introduced by (Stevenson, 1980). And if $\mu = 0$, (9) further reduces to the likelihood function for the Half Normal-Normal model introduced by (Aigner, Lovell, and Schmidt,1977).

ε is asymmetrically distributed. We have its mean as

$$
E[\varepsilon] = \frac{\sigma(\phi(\frac{\mu}{\sigma}) - \phi(\frac{B-\mu}{\sigma})) + \mu\Phi(\frac{\mu}{\sigma}) - \mu(1 - \Phi(\frac{B-\mu}{\sigma}))}{\Phi(\frac{B-\mu}{\sigma_u}) - \Phi(\frac{-\mu}{\sigma_u})} \tag{10}
$$

More importantly, we have the following conditional distribution of u given ε:

$$
f_{u|\varepsilon}(x|y) = \frac{\frac{1}{\sigma_*}\phi(\frac{x-\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})}\mathbf{1}_{[0,B]}(x), \tag{11}
$$

where

$$
\mu_* = \frac{\mu\sigma_v^2 - y\sigma_u^2}{\sigma^2} \tag{12}
$$

$$
\sigma_* = \frac{\sigma_u\sigma_v}{\sigma} \tag{13}
$$

Not surprisingly, the conditional distribution of u given ε is also doubly truncated normal with mean and variance that depend on the distribution of v as well as u.

Then we have conditional mean of u given ε,

$$
E[u|\varepsilon = y] = \mu_* + \sigma_* \frac{\phi(-\frac{\mu_*}{\sigma_*}) - \phi(\frac{B-\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})}. \tag{14}
$$

If we let $B \to \infty$, this collapses to the conditional mean from the Normal-Truncated Normal Model,

$$E[u|\varepsilon = y] = \mu_* + \sigma_* \Phi(\frac{\mu_*}{\sigma_*})^{-1} \phi(\frac{\mu_*}{\sigma_*}). \tag{15}$$

Note that individual inefficiency u_i for production unit i, as part of the composed error term ε_i, is not directly estimable. But we are able to estimate ε_i. Under our assumption of independence of u_i from $u_j, \forall j \neq i$, from $v_j, \forall j$, and from the regressors, $E[u_i|\varepsilon_i = \hat{\varepsilon}_i]$ is the best predictor of u_i given available information.

2.2 Truncated Half Normal Model

Setting $\mu = 0$, we obtain the results for the normal-truncated half normal model. The marginal distribution for ε is

$$\begin{aligned} f_\varepsilon(x) &= [\Phi(\frac{B}{\sigma_u}) - 1/2]^{-1} \cdot \frac{1}{\sigma} \phi(\frac{x}{\sigma}) \cdot \\ & \quad [\Phi(\frac{(B+x)\lambda + B\lambda^{-1}}{\sigma}) - \Phi(\frac{x\lambda}{\sigma})], \end{aligned} \tag{16}$$

where σ and λ are defined in (7), and we can also use γ-parameterization defined in (8).

The mean of ε is

$$E\varepsilon = \frac{\sigma(\frac{1}{\sqrt{2\pi}} - \phi(\frac{B}{\sigma}))}{\Phi(\frac{B}{\sigma_u}) - \frac{1}{2}} \tag{17}$$

And the conditional mean of the efficiency term is given by

$$E[u|\varepsilon = y] = \mu_* + \sigma_* \frac{\phi(-\frac{\mu_*}{\sigma_*}) - \phi(\frac{B-\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})}, \tag{18}$$

where σ_* is the same in (13) and

$$\mu_* = \frac{-y\sigma_u^2}{\sigma^2}. \tag{19}$$

2.3 Truncated Exponential Model

We next assume u has a truncated exponential distribution with the following density function,

$$f_u(x) = \frac{1}{\sigma_u(1 - e^{-B/\sigma_u})} e^{-\frac{x}{\sigma_u}} \mathbf{1}_{[0,B]}(x). \tag{20}$$

Using similar derivations as above we can derive the marginal distribution of ε,

$$f_\varepsilon(x) = \frac{e^{\frac{y}{\sigma_u}+\frac{\sigma_v^2}{2\sigma_u^2}}[\Phi(\frac{B+y}{\sigma_v}+\frac{\sigma_v}{\sigma_u}) - \Phi(\frac{y}{\sigma_v}+\frac{\sigma_v}{\sigma_u})]}{\sigma_u(1-e^{-\sigma_u/B})}. \tag{21}$$

The conditional distribution of u given ε is

$$f_{u|\varepsilon}(x|y) = \frac{\frac{1}{\sigma_v}\phi(\frac{x-\mu_*}{\sigma_v})}{\Phi(\frac{B-\mu_*}{\sigma_v}) - \Phi(-\frac{\mu_*}{\sigma_v})}\mathbf{1}_{[0,B]}(x), \tag{22}$$

where

$$\mu_* = -y - \frac{\sigma_v^2}{\sigma_u}. \tag{23}$$

The conditional mean of u is

$$E[u|\varepsilon = y] = \mu_* + \sigma_v \frac{\phi(-\frac{\mu_*}{\sigma_v}) - \phi(\frac{B-\mu_*}{\sigma_v})}{\Phi(\frac{B-\mu_*}{\sigma_v}) - \Phi(-\frac{\mu_*}{\sigma_v})}. \tag{24}$$

If $B = \infty$, this collapses to the conditional mean from the Normal-Exponential model,

$$E[u|\varepsilon = y] = \mu_* + \sigma_v \Phi(\frac{\mu_*}{\sigma_v})^{-1}\phi(\frac{\mu_*}{\sigma_v}). \tag{25}$$

2.4 The Skewness problem

A common and important methodological problem encountered when dealing with empirical implementation of the stochastic frontier model is that the residuals may be skewed in the wrong direction. In the case of the stochastic production frontier, the residuals may be positively skewed even though the composed error term $v-u$ should display negative skewness, in keeping with $u's$ positive skewness. This problem has important consequences for the interpretation of the skewness of the error term as a measure of technological inefficiency. It may imply that there had been an unfortunate sampling from an inefficiency distribution that has a negative population skewness. It may also be that positive skewness of the composed error indicates that there are no inefficiencies and that all firms are “super efficient”, a term first used by Green and Mayes (1991). The later would suggest setting the variance of inefficiency term at zero, which would have problematic impacts on estimation and on inference. Some authors have considered one-sided distributions of inefficiencies that can have negative or positive skew (Johnson et al., 1994; Carree, 2002). However, the negative skewness is also problematic since it implies that only a very small fraction of the firms attain a level of productivity close to the frontier.

The skewness of each distribution we consider in our family of doubly truncated stochastic production frontiers can be obtained from cumulants based on the moment generating function. The skewness of the *doubly truncated normal distribution* is given by

$$
\begin{aligned}
s &= -\tfrac{1}{V^{3/2}} \cdot \{2 \left(\tfrac{\phi(\frac{B-\mu_*}{\sigma_*}) - \phi(-\frac{\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})} \right)^3 + \\
& \quad 3 \left(\frac{\frac{B-\mu_*}{\sigma_*}\phi(\frac{B-\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})} - 3\frac{\mu_*}{\sigma_*} \frac{\phi(-\frac{\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})} - 1 \right) \\
\cdot & \quad \left(\frac{\phi(\frac{B-\mu_*}{\sigma_*}) - \phi(-\frac{\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})} \right) + \left(\tfrac{B-\mu_*}{\sigma_*} \right)^2 \left(\tfrac{\phi(\frac{B-\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})} \right) \\
& \quad -\tfrac{\mu_*^2}{\sigma_*^2} \left(\frac{\phi(-\frac{\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})} \right) \}
\end{aligned} \tag{26}
$$

where

$$
\begin{aligned}
V &= 1 - \left(\left(\frac{B-\mu_*}{\sigma_*} \right) \tfrac{\phi(\frac{B-\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})} + \frac{\mu_*}{\sigma_*} \frac{\phi(-\frac{\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})} \right) \\
& \quad - \left(\frac{\phi(\frac{B-\mu_*}{\sigma_*}) - \phi(-\frac{\mu_*}{\sigma_*})}{\Phi(\frac{B-\mu_*}{\sigma_*}) - \Phi(-\frac{\mu_*}{\sigma_*})} \right)^2
\end{aligned} \tag{27}
$$

is the second central moment of doubly truncated distribution , and μ_* and σ_* are defined in (12) and (13), respectively.

Letting $\mu = 0$ we obtain the skewness of the *truncated half-normal distribution*, which is the same as the expression (26) with only difference that μ_* is given by the expression in (19).

In case of *truncated exponential model*, by noting that the conditional distribution of u given ε is also the doubly truncated normal distribution, we the conditional skewness is

$$
\begin{aligned}
s \;=\; & -\tfrac{1}{V^{3/2}} \cdot \{2\left(\frac{\phi(\frac{B-\mu_*}{\sigma_v})-\phi(-\frac{\mu_*}{\sigma_v})}{\Phi(\frac{B-\mu_*}{\sigma_v})-\Phi(-\frac{\mu_*}{\sigma_v})}\right)^3 + \\
& 3\left(\frac{\frac{B-\mu_*}{\sigma_v}\phi(\frac{B-\mu_*}{\sigma_v})}{\Phi(\frac{B-\mu_*}{\sigma_v})-\Phi(-\frac{\mu_*}{\sigma_v})} - 3\frac{\mu_*}{\sigma_v}\frac{\phi(-\frac{\mu_*}{\sigma_v})}{\Phi(\frac{B-\mu_*}{\sigma_v})-\Phi(-\frac{\mu_*}{\sigma_v})} - 1\right) \\
& \cdot\left(\frac{\phi(\frac{B-\mu_*}{\sigma_v})-\phi(-\frac{\mu_*}{\sigma_v})}{\Phi(\frac{B-\mu_*}{\sigma_v})-\Phi(-\frac{\mu_*}{\sigma_v})}\right) + \left(\frac{B-\mu_*}{\sigma_v}\right)^2\left(\frac{\phi(\frac{B-\mu_*}{\sigma_v})}{\Phi(\frac{B-\mu_*}{\sigma_v})-\Phi(-\frac{\mu_*}{\sigma_v})}\right) \\
& -\frac{\mu_*^2}{\sigma_v^2}\left(\frac{\phi(-\frac{\mu_*}{\sigma_v})}{\Phi(\frac{B-\mu_*}{\sigma_v})-\Phi(-\frac{\mu_*}{\sigma_v})}\right)\}
\end{aligned}
\tag{28}
$$

where

$$\mu_* = -y - \frac{\sigma_v^2}{\sigma_u}.$$

which is the expression in (23)

Using the asymptotic expansion of the Mill's ratio for $B = \infty$, we get $s = 2$ which is the skewness of the exponential distribution. Note that in all three cases we have a positively skewed conditional distribution of u given ε and thus our new bounded inefficiency model shares this property with the classical stochastic frontier model of Aigner et al. (1977). Of course when the positively skewed inefficiency component is subtracted from the random error the composed error of the stochastic frontier is negatively skewed. Findings of positive sample skewness of the residuals of the composed error remains a finite sample issue. However, truncation of the right tail of the positively skewed inefficiency distribution reduces the level of skewness and thus the extent to which significant skewness can be revealed in samples whose inefficiencies are bounded[3]. We have examined this in more depth in a series of Monte Carlo simulations which suggest that the finding in many empirical studies using the standard one-sided unbounded inefficiency distribution of incorrect skewness may well be because the underlying distribution of inefficiency is in fact bounded. When the distribution of the one-sided inefficiency error is bounded, then standard tests for the absence of skewness are often rejected unless the variance in the one-sided

[3] It is one thing that the OLS residuals exhibit "wrong skewness"(positive skewness) and quite another that MLE estimates of λ be negative or zero. In fact, since our MLE usually uses constrained minimization, the λ estimate usually cannot be negative. We used both constrained and unconstrained minimization in our simulations below. When there is positive skewness, we find that MLE can still give reasonable estimates of λ as well as other model parameters. Moreover, when OLS residuals exhibits negative skewness, MLE may also give negative or close-to-zero values to λ.

error is large relative to the variance of the stochastic error (large λ). Thus weak identifiability of sample skewness is a property of samples in which the inefficiency distribution is bounded. We conjecture that this is the major reason many researchers have concluded the stochastic frontier model may not properly specify the inefficiency distributions in many empirical settings since the standard inefficiency distributions specified in the traditional stochastic frontier composed error are not bounded[4].

2.5 Panel Data

In the same spirit as Schmidt and Sickles (1984) and Cornwell, et al. (1990) we can specify a model for panel data:

$$y_{it} = A + \sum_{k=1}^{K} \alpha_k x_{it,k} + \varepsilon_{it} \tag{29}$$

where

$$\varepsilon_{it} = v_{it} - u_{it}. \tag{30}$$

We keep the assumption that the inefficiency components u_{it} are iid and are independent from the regressors, and we assume u_{it} to have upper bound B_t for each t. We may set B_t to be time-invariant. However, it is certainly more plausible to assume otherwise, as the market or industry may well become more or less forgiving as time goes by, especially in settings in which market reforms are being introduced or firms are adjusting to a phased transition from regulation to deregulation.

There are two possible extensions to this specification. First, we may drop the assumption that u is independent from the regressors. In the panel data setting, this assumption is more than necessary for the identification of the model. In fact, here we treat panel data merely as a collection of cross-section data in the chronological order. The advantage of panel data is not exploited. Second, we may certainly give more structure to B_t. For example, B_t could be specified as $B_t = \sum_{i=1}^{m} \beta_i f_t$ for some appropriate class of functions (f_t). We leave these two possibilities for future investigations.

3 Estimation

With the distribution of the error term fully specified, maximum likelihood is a natural estimator to use in these models. Here we list the log-likelihood functions of these

[4] We thank C. A. Knox Lovell for his observation, which he made at the Tenth European Workshop on Efficiency and Productivity, Lille, France, June, 2007, that there was potential for our bounded frontier to address the skewness problem inherent in the use of the Aigner, Lovell and Schmidt stochastic frontier model

models. Note that in practice we need also provide analytic form of the gradient of the log likelihood function. The gradients are complicated in form but straightforward to derive and we omit them here.

The log-likelihood function for the doubly truncated normal model with λ parameterization is given by

$$
\begin{aligned}
\log L &= -n \log \left[\Phi(\frac{B-\mu}{\sigma_u(\sigma,\lambda)}) - \Phi(\frac{-\mu}{\sigma_u(\sigma,\lambda)}) \right] \\
&\quad -n \log \sigma - \frac{n}{2}\log(2\pi) - \sum_{i=1}^{n} \frac{(\varepsilon_i+\mu)^2}{2\sigma^2} \\
&\quad + \sum_{i=1}^{n} \log\{\Phi(\frac{(B+\varepsilon_i)\lambda + (B-\mu)\lambda^{-1}}{\sigma}) \\
&\qquad\qquad - \Phi(\frac{\varepsilon_i\lambda - \mu\lambda^{-1}}{\sigma})\},
\end{aligned} \tag{31}
$$

where $\varepsilon_i = y - A - \sum x_{i,k}\alpha_k$ and

$$
\sigma_u(\sigma,\lambda) = \frac{\sigma}{\sqrt{1+1/\lambda^2}}. \tag{32}
$$

It is easy to get $\log L$ in γ-parameterization. We can substitute λ in (31) with

$$
\lambda(\gamma) = \sqrt{\frac{\gamma}{1-\gamma}}. \tag{33}
$$

The log-likelihood function for the truncated half normal model is

$$
\begin{aligned}
\log L &= -n \log(\Phi(\frac{B}{\sigma_u(\sigma,\lambda)}) - \frac{1}{2}) \\
&\quad -n \log \sigma - \frac{n}{2}\log(2\pi) - \sum_{i=1}^{n} \frac{\varepsilon_i^2}{2\sigma^2} \\
&\quad + \sum_{i=1}^{n} \log\{\Phi(\frac{(B+\varepsilon_i)\lambda + B\lambda^{-1}}{\sigma}) - \Phi(\frac{\varepsilon_i\lambda}{\sigma})\},
\end{aligned} \tag{34}
$$

where $\varepsilon_i = y - A - \sum x_{i,k}\alpha_k$ and $\sigma_u(\sigma,\lambda)$ is defined in (32). Again, substitute λ in (34) with $\lambda(\gamma)$ in (33), we get $\log L$ of γ-parameterization.

Finally, the log-likelihood function for the truncated exponential model is

$$
\begin{aligned}
\log L &= -n \log \sigma_u - n \log(1 - e^{-\sigma_u/B}) \\
&\quad + \frac{n\sigma_v^2}{2\sigma_u^2} + \frac{1}{\sigma_u}\sum_{i=1}^{n} \varepsilon_i && (35) \\
&\quad + \sum_{i=1}^{n} \log[\Phi(\frac{B+\varepsilon_i}{\sigma_v} + \frac{\sigma_v}{\sigma_u}) - \Phi(\frac{\varepsilon_i}{\sigma_v} + \frac{\sigma_v}{\sigma_u})], && (36)
\end{aligned}
$$

where $\varepsilon_i = y - A - \sum x_{i,k}\alpha_k$.

After estimating the model, we can estimate the composed error term ε_i:

$$\hat{\varepsilon}_i = y_i - \hat{A} - \sum x_{i,k}\hat{\alpha}_k, i = 1, \cdots, n. \tag{37}$$

From this we can estimate the inefficiency term u_i using the formula for $E[u|\varepsilon = \hat{\varepsilon}_i]$ in (14), (18), and (24) for doubly truncated, truncated half normal, and truncated exponential models, respectively.

4 Monte Carlo Results

To examine the finite sample performance of the MLE estimators we run a series of Monte Carlo experiments for the standard cross-sectional stochastic frontier model. The data generating process is (1) and (2) with $A = 0$ and $K = 2$ (two regressors and no constant term). Throughout we set $\alpha_1 = 0.6$, $\alpha_2 = 0.5$. We set $\sigma_u = 1.0$ in truncated half normal model and doubly truncated normal model, and $\sigma_u = 0.3$ in truncated exponential model. To examine how the noise level (σ_v) affects the quality of estimation, we vary σ_v from 0.1, 0.2, to 0.5. In the other dimension, we change the inefficiency bound from 0.8, 1.0, to 1.2, to examine its impact on estimation. For both truncated half normal and doubly truncated normal models we use the γ-parameterization, and thus the parameters to be estimated are σ and γ as well as the production parameters.

Table 1 and 2 reports results from the truncated half normal model with a sample size of 200 and 500, respectively. The results from these two tables differ only in quantitative manner.

The first important conclusion that can be drawn from Table 1 and 2 is that the MLE estimators for technology parameters, α_1 and α_2, are accurate. As noise level increases, the MSE of these estimators only slightly increases. The second important observation is that the estimator for the inefficiency bound is has small MSE when the noise level is mild. When noise level is high, as when $\sigma_v = 0.5$, $\hat{B}$ becomes inaccurate.

For the distribution parameters, $\hat{\sigma}$ displays a significantly upward bias, a large MSE, and sensitivity to the noise level. The problem is alleviated somewhat when the inefficiency bound B becomes higher. In all cases $\hat{\gamma}$ is unbiased and has small MSE.

We now look at the doubly truncated normal model. Table 3 and 4 reports Monte Carlo results with a sample size of 200 and 500, respectively. At the sample size 200, the technology parameter estimates $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are downward biased and have large MSE. Interestingly, they become less biased and have smaller MSE's as the noise level σ_v increases. The inefficiency bound estimate $\hat{B}$ is also inaccurate and sensitive to the noise level. The estimation for distribution parameters, μ, σ, and γ, are poor, especially when B is small. This indicates that when sample size is small, the effects

Table 1: Monte Carlo results for Truncated Half Normal model. The number of repetitions $M = 1000$. Sample size $N = 200$.

			$B = 0.8$		$B = 1.0$		$B = 1.2$	
		True	AVE	MSE	AVE	MSE	AVE	MSE
	$\hat{\sigma}$	1.005	1.461	0.996	1.248	0.460	1.151	0.273
	$\hat{\gamma}$	0.990	0.988	0.001	0.987	0.004	0.980	0.011
$\sigma_v = 0.1$	$\hat{B}$		0.806	0.004	1.001	0.008	1.188	0.020
	$\hat{\alpha}_1$	0.6	0.600	0.002	0.600	0.004	0.595	0.007
	$\hat{\alpha}_2$	0.5	0.499	0.002	0.498	0.003	0.493	0.005
	$\hat{\sigma}$	1.020	1.304	0.645	1.380	0.774	1.284	0.522
	$\hat{\gamma}$	0.962	0.946	0.004	0.954	0.005	0.961	0.001
$\sigma_v = 0.2$	$\hat{B}$		0.803	0.036	1.019	0.020	1.220	0.015
	$\hat{\alpha}_1$	0.6	0.598	0.004	0.596	0.005	0.602	0.007
	$\hat{\alpha}_2$	0.5	0.493	0.004	0.502	0.004	0.500	0.005
	$\hat{\sigma}$	1.118	1.370	0.495	1.423	0.598	1.532	0.859
	$\hat{\gamma}$	0.800	0.827	0.010	0.826	0.013	0.823	0.021
$\sigma_v = 0.5$	$\hat{B}$		0.901	0.402	1.132	0.418	1.310	0.417
	$\hat{\alpha}_1$	0.6	0.596	0.017	0.600	0.017	0.599	0.019
	$\hat{\alpha}_2$	0.5	0.502	0.015	0.502	0.016	0.497	0.018

Table 2: Monte Carlo results for Truncated Half Normal model. The number of repetitions $M = 1000$. Sample size $N = 500$.

			$B = 0.8$		$B = 1.0$		$B = 1.2$	
		True	AVE	MSE	AVE	MSE	AVE	MSE
$\sigma_v = 0.1$	$\hat{\sigma}$	1.005	1.244	0.477	1.112	0.136	1.046	0.050
	$\hat{\gamma}$	0.990	0.990	0.000	0.990	0.000	0.990	0.000
	$\hat{B}$		0.804	0.001	1.000	0.001	1.202	0.002
	$\hat{\alpha}_1$	0.6	0.600	0.001	0.601	0.001	0.601	0.001
	$\hat{\alpha}_2$	0.5	0.500	0.000	0.500	0.001	0.499	0.001
$\sigma_v = 0.2$	$\hat{\sigma}$	1.020	1.201	0.404	1.206	0.315	1.130	0.170
	$\hat{\gamma}$	0.962	0.949	0.002	0.959	0.001	0.961	0.001
	$\hat{B}$		0.804	0.011	1.007	0.006	1.208	0.005
	$\hat{\alpha}_1$	0.6	0.597	0.001	0.598	0.002	0.603	0.002
	$\hat{\alpha}_2$	0.5	0.499	0.001	0.502	0.001	0.498	0.001
$\sigma_v = 0.5$	$\hat{\sigma}$	1.118	1.217	0.140	1.273	0.257	1.320	0.349
	$\hat{\gamma}$	0.800	0.809	0.007	0.804	0.011	0.800	0.017
	$\hat{B}$		0.841	0.187	1.067	0.239	1.290	0.250
	$\hat{\alpha}_1$	0.6	0.598	0.006	0.598	0.007	0.595	0.007
	$\hat{\alpha}_2$	0.5	0.498	0.006	0.495	0.006	0.497	0.005

Table 3: Monte Carlo results for Doubly Truncated Normal model. The number of repetitions $M = 1000$. Sample size $N = 200$.

			$B = 0.8$		$B = 1.0$		$B = 1.2$	
		True	AVE	MSE	AVE	MSE	AVE	MSE
	$\hat{\sigma}$	1.005	0.760	0.351	0.811	0.270	0.789	0.236
	$\hat{\gamma}$	0.990	0.856	0.125	0.896	0.089	0.867	0.120
$\sigma_v = 0.1$	$\hat{\mu}$	0	0.295	0.325	0.229	0.185	0.166	0.078
	$\hat{B}$		0.732	0.088	0.943	0.098	1.081	0.182
	$\hat{\alpha}_1$	0.6	0.527	0.047	0.550	0.035	0.534	0.046
	$\hat{\alpha}_2$	0.5	0.443	0.033	0.459	0.024	0.440	0.032
	$\hat{\sigma}$	1.020	0.763	0.274	0.776	0.276	0.814	0.250
	$\hat{\gamma}$	0.962	0.830	0.091	0.836	0.098	0.849	0.096
$\sigma_v = 0.2$	$\hat{\mu}$	0	0.619	1.273	0.352	0.422	0.264	0.221
	$\hat{B}$		0.806	0.136	0.990	0.156	1.170	0.195
	$\hat{\alpha}_1$	0.6	0.559	0.034	0.551	0.040	0.551	0.041
	$\hat{\alpha}_2$	0.5	0.470	0.025	0.462	0.028	0.460	0.028
	$\hat{\sigma}$	1.118	1.102	0.145	1.087	0.162	1.096	0.231
	$\hat{\gamma}$	0.800	0.778	0.022	0.759	0.033	0.737	0.051
$\sigma_v = 0.5$	$\hat{\mu}$	0	0.917	2.594	0.940	2.485	0.887	2.275
	$\hat{B}$		0.901	0.662	1.110	0.711	1.337	0.727
	$\hat{\alpha}_1$	0.6	0.612	0.023	0.612	0.028	0.602	0.031
	$\hat{\alpha}_2$	0.5	0.515	0.020	0.511	0.025	0.511	0.029

of these distribution parameters are difficult to be disentangled from that of B. At the sample size of 500, all problems above disappear except that of μ. μ remains inaccurate and sensitive to the noise level.

Table 5 and 6 show the results for truncated exponential model with a sample size of 200 and 500, respectively. As with the truncated half normal case, the technology parameter estimates, $\hat{\alpha}_1$ and $\hat{\alpha}_2$, are accurate, and the inefficiency bound estimate, $\hat{B}$, is accurate when the noise level is mild. For the distribution parameters, when the noise level is low ($\sigma_v = 0.1$), we observe only a slight upward bias in $\hat{\sigma}_u$ and downward bias in $\hat{\sigma}_v$. When the noise level is medium or high, however, the performance of $\hat{\sigma}_u$ is rather poor. The more so, when the inefficiency bound B is also low.

As expected, the finite sample problem with $\hat{\sigma}_u$ is lessened when we have a larger sample size 500. Table 6 shows that $\hat{\sigma}_u$ is acceptable when the noise level is medium

Table 4: Monte Carlo results for Doubly Truncated Normal model. The number of repetitions $M = 1000$. Sample size $N = 500$.

			$B = 0.8$		$B = 1.0$		$B = 1.2$	
		True	AVE	MSE	AVE	MSE	AVE	MSE
	$\hat{\sigma}$	1.005	0.812	0.192	0.836	0.123	0.888	0.075
	$\hat{\gamma}$	0.990	0.974	0.006	0.983	0.002	0.985	0.003
$\sigma_v = 0.1$	$\hat{\mu}$	0	0.205	0.095	0.187	0.076	0.160	0.062
	$\hat{B}$		0.828	0.008	1.028	0.006	1.220	0.008
	$\hat{\alpha}_1$	0.6	0.599	0.003	0.602	0.001	0.600	0.002
	$\hat{\alpha}_2$	0.5	0.504	0.002	0.505	0.001	0.503	0.001
	$\hat{\sigma}$	1.020	0.822	0.158	0.833	0.148	0.870	0.110
	$\hat{\gamma}$	0.962	0.901	0.018	0.921	0.013	0.940	0.007
$\sigma_v = 0.2$	$\hat{\mu}$	0	0.396	0.433	0.258	0.161	0.233	0.127
	$\hat{B}$		0.869	0.043	1.076	0.042	1.276	0.040
	$\hat{\alpha}_1$	0.6	0.606	0.003	0.603	0.005	0.606	0.004
	$\hat{\alpha}_2$	0.5	0.508	0.003	0.509	0.004	0.509	0.003
	$\hat{\sigma}$	1.118	1.058	0.051	1.060	0.085	1.044	0.094
	$\hat{\gamma}$	0.800	0.757	0.015	0.743	0.023	0.726	0.030
$\sigma_v = 0.5$	$\hat{\mu}$	0	0.698	1.418	0.713	1.366	0.628	1.054
	$\hat{B}$		0.838	0.246	1.089	0.292	1.353	0.408
	$\hat{\alpha}_1$	0.6	0.606	0.007	0.613	0.008	0.614	0.007
	$\hat{\alpha}_2$	0.5	0.512	0.007	0.516	0.008	0.517	0.009

Table 5: Monte Carlo results for Truncated Exponential model. The number of repetitions $M = 1000$. Sample size $N = 200$.

		True	$B = 0.8$		$B = 1.0$		$B = 1.2$	
			AVE	MSE	AVE	MSE	AVE	MSE
	$\hat{\sigma}_u$	0.3	0.3192	0.0053	0.3127	0.0029	0.3103	0.0019
	$\hat{\sigma}_v$	0.1	0.0950	0.0003	0.0952	0.0003	0.0956	0.0003
$\sigma_v = 0.1$	$\hat{B}$		0.8063	0.0046	0.9928	0.0074	1.1847	0.0125
	$\hat{\alpha}_1$	0.6	0.6010	0.0010	0.5994	0.0012	0.5989	0.0012
	$\hat{\alpha}_2$	0.5	0.5011	0.0007	0.5015	0.0008	0.5020	0.0008
	$\hat{\sigma}_u$	0.3	0.9429	20.7714	0.3792	0.2352	0.3557	0.5932
	$\hat{\sigma}_v$	0.2	0.1912	0.0010	0.1911	0.0009	0.1926	0.0008
$\sigma_v = 0.2$	$\hat{B}$		0.8376	0.0344	1.0311	0.0375	1.2145	0.0420
	$\hat{\alpha}_1$	0.6	0.6032	0.0025	0.5988	0.0026	0.6005	0.003
	$\hat{\alpha}_2$	0.5	0.5039	0.0019	0.5071	0.0021	0.5033	0.0020
	$\hat{\sigma}_u$	0.3	3.1291	79.5727	2.7975	57.8804	2.1373	44.1386
	$\hat{\sigma}_v$	0.5	0.4652	0.0054	0.4614	0.0062	0.4656	0.0059
$\sigma_v = 0.5$	$\hat{B}$		1.0297	0.4266	1.2061	0.4327	1.3278	0.4284
	$\hat{\alpha}_1$	0.6	0.6260	0.0118	0.6226	0.0111	0.6260	0.0121
	$\hat{\alpha}_2$	0.5	0.5175	0.0093	0.5274	0.0093	0.5203	0.0100

Table 6: Monte Carlo results for Truncated Exponential model. The number of repetitions $M = 1000$. Sample size $N = 500$.

			$B = 0.8$		$B = 1.0$		$B = 1.2$	
		True	AVE	MSE	AVE	MSE	AVE	MSE
	$\hat{\sigma}_u$	0.3	0.3054	0.0016	0.3041	0.0009	0.3034	0.0006
	$\hat{\sigma}_v$	0.1	0.0985	0.0001	0.0980	0.0001	0.0979	0.0001
$\sigma_v = 0.1$	$\hat{B}$		0.8017	0.0018	0.9970	0.0029	1.1929	0.0052
	$\hat{\alpha}_1$	0.6	0.5999	0.0005	0.6001	0.0005	0.6003	0.0006
	$\hat{\alpha}_2$	0.5	0.5005	0.0004	0.5002	0.0004	0.5004	0.0004
	$\hat{\sigma}_u$	0.3	0.3481	0.0592	0.3196	0.0059	0.3093	0.0022
	$\hat{\sigma}_v$	0.2	0.1960	0.0004	0.1956	0.0004	0.1974	0.0003
$\sigma_v = 0.2$	$\hat{B}$		0.8245	0.0153	1.0193	0.0130	1.2155	0.0207
	$\hat{\alpha}_1$	0.6	0.6027	0.0012	0.6014	0.0012	0.5992	0.001
	$\hat{\alpha}_2$	0.5	0.5012	0.0009	0.5011	0.0009	0.5023	0.0009
	$\hat{\sigma}_u$	0.3	1.3566	9.7584	1.1411	5.2664	0.7721	2.2743
	$\hat{\sigma}_v$	0.5	0.4826	0.0022	0.4829	0.0023	0.4872	0.0023
$\sigma_v = 0.5$	$\hat{B}$		0.9572	0.3130	1.1645	0.3353	1.3228	0.3847
	$\hat{\alpha}_1$	0.6	0.6052	0.0058	0.6108	0.0061	0.6095	0.0054
	$\hat{\alpha}_2$	0.5	0.5162	0.0051	0.5160	0.0051	0.5108	0.0048

($\sigma_v = 0.2$). When the noise level is high, however, the problem remains. Other parameter estimates exhibit high finite sample qualities.

5 Efficiency Analysis of Banking Industry

5.1 Empirical Model and Data

We now apply the bounded inefficiency (BIE) model to an analysis of the US banking industry, which underwent a series of deregulatory reforms in the early 1980's. Here we extend our model to the pane setting and, following Adams, Berger, and Sickles (1999) and Kneip, Sickles, and Song(2005), we specify a Cobb-Douglas stochastic distance frontier model as follows,

$$Y_{it} = Y_{it}^{*\prime}\gamma + X_{it}'\beta + v_{it} - u_{it}, \tag{38}$$

where Y_{it} is real estate loans; X_{it} includes certificate of deposit (CD), demand deposit (DD), retail time/savings deposit (OD), labor (lab), capital (cap), and purchased funds (purf); and Y_{it}^* includes commercial and industrial loans/real estate loans (ciln) and installment loans/real estate loans (inln). All X_{it} and Y_{it}^* are transformed by $-\log(\cdot)$. We assume (v_{it}) are iid across i and t, and for each t, u_{it} has a upper bound B_t. Then we can treat this model as a generic panel data bounded inefficiency model as discussed in Section 2.5. Once the individual effects u_{it} are estimated, technical efficiency for a particular firm at time t is calculated as $TE = \exp(u_{it} - \max_{1\leq j\leq N} u_{jt})$.

We use US commercial banking panel data from 1984 through 1995 in limited branching regulatory environment. The data are taken from the Report of Condition and Income (Call Report) and the FDIC Summary of Deposits. The data set include 8004 observations for 667 banks. For more detailed discussion, readers are referred to the Appendix of (Jayasiriya, 2000).

We compare the BIE estimator to the "Within" estimator (Schmidt and Sickles, 1984), CSSW (Cornwell, Schmidt, and Sickles, 1990), KSS (Kneip, Sickles, and Song, 2005), and BC (Battese and Coelli, 1992).

5.2 Results

Table 7 compares the parameter estimates of the bounded inefficiency (BIE) with that of "Within", CSS, KSS, and BC. The BIE estimates are generally different from previous results. The parameter estimates for DD (direct demand) and cap (capital) are markedly larger than previously estimated, while the estimate for purf (purchased funds) is much lower than previous results.

An even more striking difference is in the average efficiency score in 12 years. The BIE average efficiency is significantly higher than what previous models obtains. This is not unexpected, however, since the existence of inefficiency bound expands the domain of the noise component v, hence attributes more of "very inefficient" firms to bad measurement and other unaccountable factors.

Of course, for time-varying efficiency models such as CSS, KSS, BC, and BIE, the average efficiency changes over time. This is illustrated in Figure 1. While it is obvious that BIE yields a higher average efficiency curve than the other models do, it traces a similar upward trend in the twelve years in terms of average efficiency. We also look at the efficiency ranking of firms. Table 8 tabulates the Spearman rank correlations among different models. It is clear that the BIE efficiency ranking is in agreement with previous estimations, especially with CSSW.

We next look at the performance (relative to the best) of the least efficient bank over the years. In Figure 2, the curves labeled "BIE", "CSSW", and "KSS" are the minimum (among all banks) efficiency scores calculated from each model for each

Table 7: Comparisons of Various Estimators. MLE estimates and standard deviations (in parentheses) for each model parameters from competing models (Within, CSSW, KSS, BC, BIE). CD: certificate of deposit; DD: demand deposit; OD: retail time/savings deposit; lab: labor; cap: capital; purf: purchased funds; ATE: Average Technical Efficiency.

	Within	CSSW	KSS	BC	BIE
CD	-0.0351 (0.0047)	-0.0099 (0.0032)	-0.0019 (0.0019)	-0.0320 (0.0044)	-0.0924 (0.0050)
DD	-0.0904 (0.0160)	-0.0813 (0.0138)	-0.0193 (0.0109)	-0.0351 (0.0138)	-0.1314 (0.0115)
OD	-0.1525 (0.0097)	-0.1245 (0.0071)	-0.0306 (0.0201)	-0.1474 (0.0090)	-0.1322 (0.0146)
lab	-0.1786 (0.0171)	-0.1508 (0.0146)	-0.0913 (0.0095)	-0.1557 (0.0147)	-0.1158 (0.0132)
cap	-0.0427 (0.0054)	-0.0458 (0.0054)	-0.0250 (0.0052)	-0.0502 (0.0048)	-0.1157 (0.0052)
purf	-0.5855 (0.0215)	-0.5263 (0.0195)	-0.5751 (0.0299)	-0.6243 (0.0195)	-0.4024 (0.0163)
ciln	0.1603 (0.0045)	0.1470 (0.0037)	0.1193 (0.0030)	0.1601 (0.0042)	0.2818 (0.0043)
inln	0.3712 (0.0061)	0.3516 (0.0056)	0.3243 (0.0049)	0.3622 (0.0055)	0.2739 (0.0058)
time	0.0145 (0.0009)	.	.	0.0016 (0.0013)	.
ATE	0.4389	0.6230	0.6027	0.6011	0.8027

Table 8: Spearman Rank Correlations of Efficiencies

	Within	CSSW	KSS	BC	BIE
Within	1	·	·	·	·
CSSW	0.8743	1	·	·	·
KSS	0.7667	0.8974	1	·	·
BC	0.9854	0.8785	0.7937	1	·
BIE	0.7607	0.8493	0.7686	0.7585	1

year. We may call these curves "minimum efficiency curves". And the curve labeled "BIE Bound" traces the inefficiency bound calculated from the BIE model over the years, that is, $\hat{B}_t$. It is worth emphasizing that the inefficiency bound is a statistical parameter that clearly sets the minimum level of inefficiency, while the points on the minimum efficiency curves are efficiency scores calculated for particular (worst performed) banks. So the curves "BIE" and "BIE Bound" differ.

A downward trend is observed for the BIE minimum efficiency curve. A similar trend, although less marked, is observed in the CSSW minimum efficiency curve. The inefficiency bound is slightly below the minimum efficiency curve for BIE and the two move in tandem. Figures 1 and 2 display an interesting finding: on one hand, an upward trend is observed for the average efficiency of the industry, presumably benefiting from the deregulations in the 1980s; on the other hand, the industry appears to be more "tolerant" of less efficient banks. Possibly, these banks have a characteristic that we have not properly controlled for and we are currently examining this issue. Given the recent experiences in the credit markets due in part to the poor oversight lending authorities gave in their mortgage and other lending activities, our results also may be indicative of a backsliding in the toleration of inefficiency that could have contributed to problems the financial services industry faces today.

6 Conclusions

In this paper we have introduced a series of parametric stochastic frontier models that have upper (lower) bounds on the inefficiency (efficiency). The model parameters can be estimated by maximum likelihood, including the inefficiency bound. The models are easily applicable for both cross-section and panel data setting. In the panel data setting, we set the inefficiency bound to be varying over time, hence contributing another time-varying efficiency model to the literature. We have examined the finite sample performance of the maximum likelihood estimator in the cross-sectional setting. An empirical analysis of US banking industry using the new model yields different estimates for some technology parameters and a considerable higher average efficiency score than previous models, and because the efficiencies are higher, one that may more easily be defended on economic grounds than the substantially lower efficiencies predicted by the classical stochastic frontier model.

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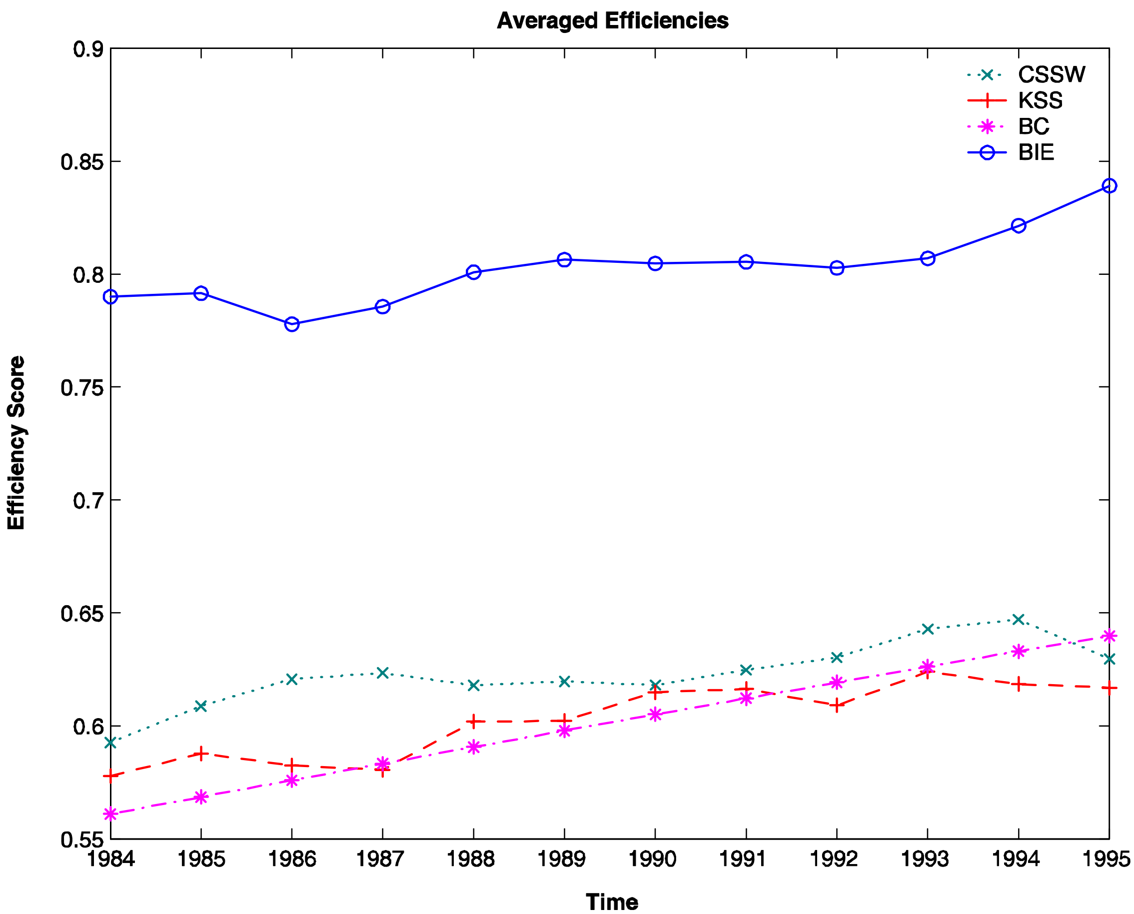

Averaged Efficiencies
CSSW
KSS
BC
BIE
Efficiency Score
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
Time

Figure 1:

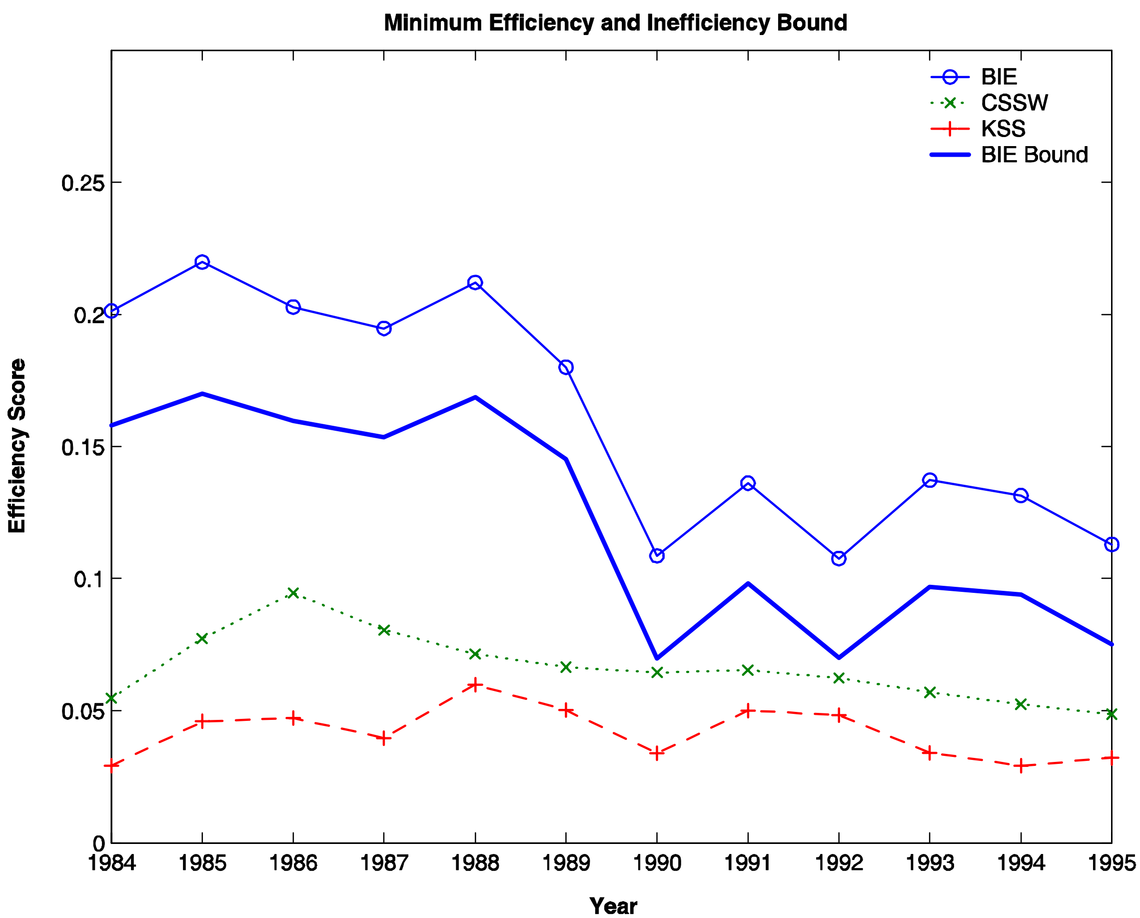
Minimum Efficiency and Inefficiency Bound
BIE
CSSW
KSS
BIE Bound
0.25
0.2
0.15
0.1
0.05
0
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
Efficiency Score
Year

Figure 2: