Chapter 5 Economic Growth

Introduction

Economic growth, or economic development, is no doubt one of the most important topics in macroeconomics. For poor countries, a stagnant economy means a persistent *absolute* poverty. In absolute poverty, the need for survival dominates all other desires of human being. As recent history tells us, human lives in absolute poverty can be extremely miserable and dangerous.

In relative terms, a slight but persistent difference in growth rate would result in huge income gaps among nations. The following table illustrates how three different growth rates of income per capita (from the same level, say 100) lead to starkly different outcomes many (10, 30, 100) years later.

Years	0	10	30	100
1%	100	110.5	134.8	270.5
3%	100	134.4	242.7	1921.9
8%	100	215.9	1006.3	219976.1

Economic growth is important not only in terms of the ultimate outcome (that is, a wealthy society), but also the path that leads to the outcome. A growing economy is itself good. People in a growing economy tend to be more optimistic toward the future. They tend to be more open and tolerant, because the pie is getting bigger. Even a wealthy nation, if it stops growing, can fell to the prey of intolerance and hostility, because people are trapped in a zero-sum game.

In this chapter we first introduce a few models that are useful for analyzing economic growth. The spot light, of course, is on the celebrated Solow models. Based on the Solow models, we study economic growth accounting. At the end of the chapter we also discuss some important topics that are related to economic growth. Note that since we are studying long-term growth and that prices are flexible in the long run, economic growth is necessarily a story about the supply side. That is, how factor inputs and technology drive the economy forward. In this sense, we are now back to the classical tradition.

Solow Model I

The first Solow model characterizes the role of factor inputs in economic growth. We assume that prices and wage are flexible, so that all factor resources (e.g., labor and capital) are fully utilized in production. This is a reasonable assumption, since we are studying the long-term trajectory of economic development. Business cycles can be seen as short-term fluctuations around the long-term trend of growth. Furthermore, we assume the following

- Closed economy ($NX = 0$), no government spending ($G = 0$).
- Fixed production function, $Y = F(K, L)$, which is a constant-return-to-scale technology.
- The saving rate is constant: $C = (1 - s)Y, 0 \leq s \leq 1$.
- Capital depreciates at a constant rate δ.
- Population grows at a constant rate n, $L_t = L_0 e^{nt}$.

Let $y = \frac{Y}{L}$ and $k = \frac{K}{L}$. Obviously, y is the output per capita and k is the amount of capital per cap. We have

$$y = \frac{Y}{L} = \frac{F(K, L)}{L} = F(k, 1).$$

We define $f(k) \equiv F(k, 1)$. Then we have

$$y = f(k).$$

f(k) is the "per worker production function". That is, how much output one worker could produce using *k* units of capital. We assume

$$f(0) = 0,\ f'(k) > 0,\ f''(k) < 0.$$

Note that $f'(k)$ is the marginal product of capital (MPK). We also assume that

$$\lim_{k \to 0} f'(k) = \infty,\ \lim_{k \to \infty} f'(k) = 0.$$

Without government spending and net export, the demand for goods and services is composed of consumption (C) and investment (I). In per cap terms, we have

$$y = c + i,$$

where $c = C/L$ and $i = \frac{I}{L}$. The per cap investment is a constant fraction of the out

$$i = y - c = y - (1 - s)y = sy.$$

Investment causes capital to rise and depreciation causes capital to wear out. The aggregate capital accumulation is described by

$$\dot{K_t} \equiv \frac{dK_t}{dt} = sF(K_t, L_t) - \delta K_t.$$

Similarly, the evolution of the population is characterized by

$$\dot{L_t} = nL_t.$$

Let $k_t = \frac{K_t}{L_t}$, the capital per cap at time t. We then characterize the per cap capital accumulation by

$$\dot{k_t} \equiv \frac{d}{dt}\left(\frac{K_t}{L_t}\right) = \frac{\dot{K_t}}{L_t} - \frac{K_t \dot{L_t}}{L_t^2} = sf(k_t) - (\delta + n)k_t.$$

Obviously, saving increases the per cap capital (k_t), depreciation decreases k_t, and population growth dilutes k_t.

As capital accumulates, it will reach a point where new investment equals depreciation and dilution by population growth,

$$i^* = sf(k^*) = (\delta + n)k^*.$$

At this level of capital, k^*, the economy reaches a steady state, where capital does not increase or decrease. We call k^* the stead-state level of capital. Note that population growth has a similar effect on steady-state capital stock with depreciation, as both reduce per capita capital stock.

Figure 1: A Graphical Illustration

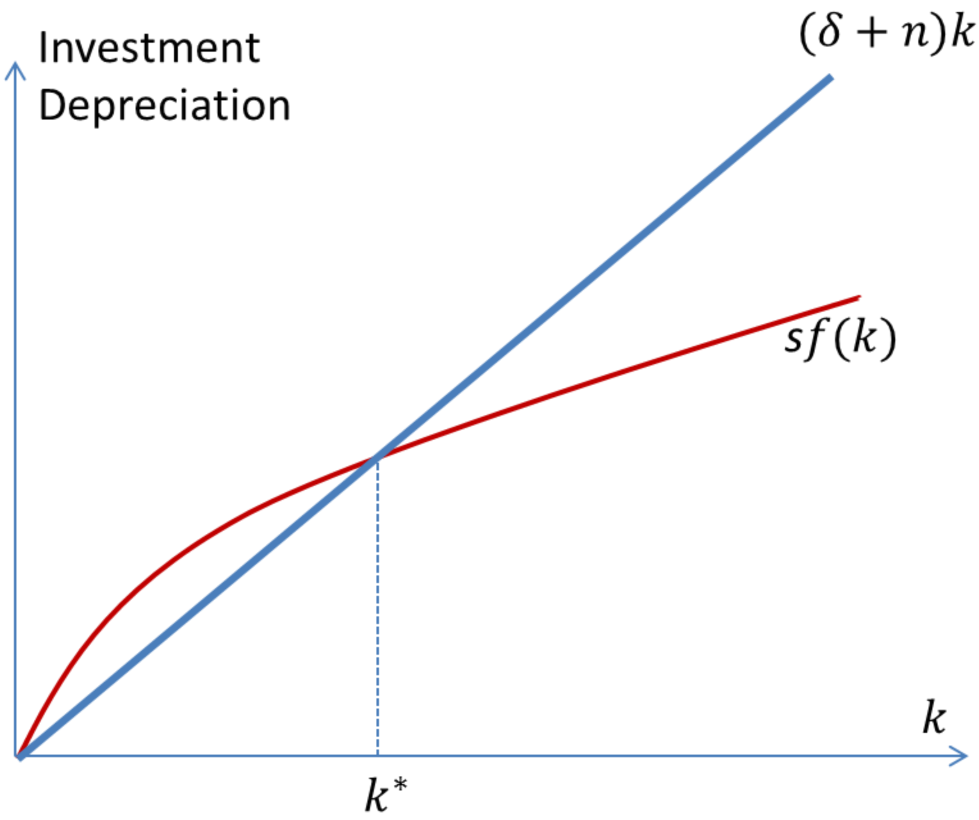


Figure 1 show how steady state is reached and maintained. If k_t is below k^*, investment ($sf(k_t)$, red line) is higher than the depreciation and the dilution of population, then the capital stock per capita would increase. If k_t is above k^*, investment ($sf(k_t)$, red line) is lower than the depreciation and the dilution of population, then the capital stock per capita would decrease. The steady state is thus maintained.

To see an example, suppose that $F(K,L) = K^{1/2}L^{1/2}$. Then we have

$$y = \frac{Y}{L} = \frac{K^{\frac{1}{2}}L^{\frac{1}{2}}}{L} = \left(\frac{K}{L}\right)^{\frac{1}{2}} = k^{1/2}.$$

Let $n = 0, s = 0.3,\ \delta = 0.1,\ k_0 = 4.$ Each year ($\Delta t = 1$), the capital stock changes by

$\Delta k = 0.3k_t^{\frac{1}{2}} - 0.1k_t$. Solving $0.3k^{*\frac{1}{2}} = 0.1k^*$, we obtain $k^* = 9$.

We can use the Excel Spreadsheet (Solow1.xlsx) to see how the economy reaches the steady state.

If the economy is already at a steady state, per capita income ($y^* = f(k^*)$) ceases to grow. However, the total income continues to grow as the population grows,

$$Y_t = y^*L_t = y^*L_0e^{nt}.$$

If the initial level of capital is below the steady-state level, there will be a convergence (or, catch-up) period to the steady-state level.

The first Solow model depicts a dismal picture of our economy. Under the assumption that the technology is constant, the income per capita would cease to grow in the steady state (which should be understood as the normal case in the Solow I economy), even though the total income could still grow with the population.

To see the effect of a change in the saving rate, s, we examine the equation characterizing the steady state,

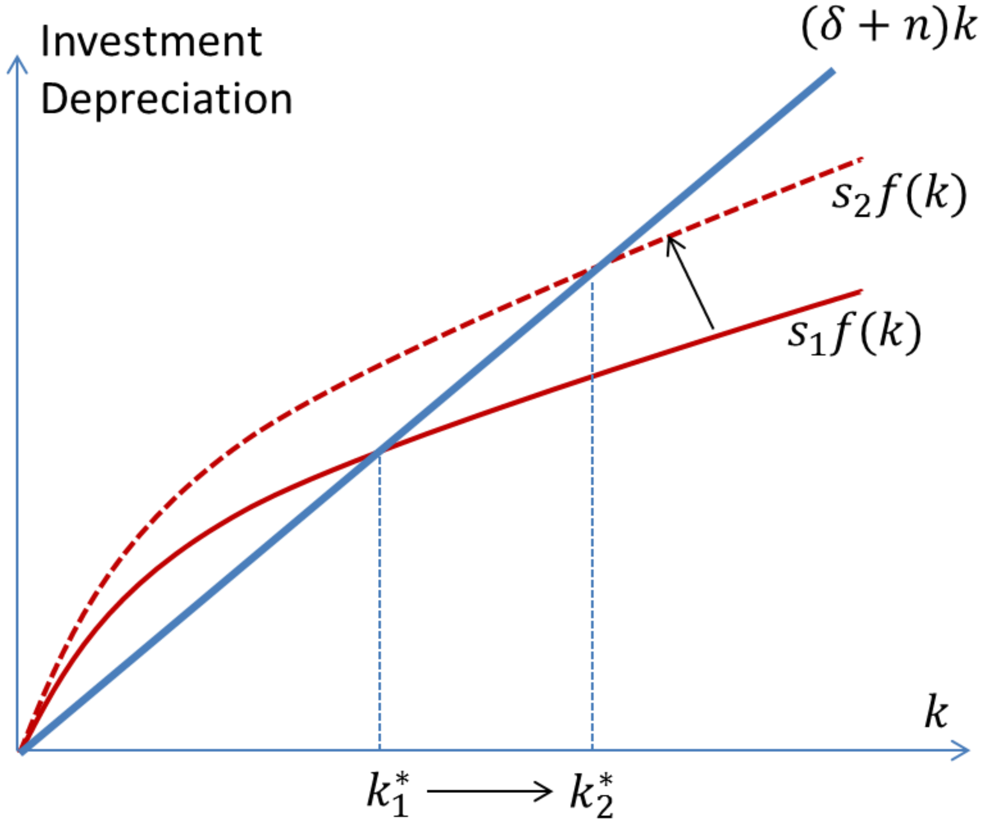
$$sf(k^*) = (\delta + n)k^*.$$

Fixing δ and n and using the implicit function theorem, we have

$$\frac{dk^*}{ds} = -\frac{f(k^*)}{sf'(k^*) - (\delta + n)}.$$

Due to the assumption $\lim_{k\to 0} f'(k) = \infty$, $\lim_{k\to\infty} f'(k) = 0$, we must have $sf'(k^*) < \delta + n$, otherwise the curve $sf(k)$ cannot cross with the line $(\delta + n)k$ at k^*. Hence $\frac{dk^*}{ds}$ must be positive. An increase in saving rate would lead to a higher level of steady-state capital and income (See Figure 2). However, once the new steady-state level of income is reached, the income per capita again stagnates.

Figure 2: The Effect of An Change in Saving Rate



If the saving rate is zero, the corresponding steady-state capital, income, and consumption would all be zero. And if the saving rate is one, then there would be nothing left for consumption. Hence too little saving and too much saving would be both undesirable. There must be some saving rate that achieves a maximum level of consumption in the stead state.

At steady states, the consumption is given by

$$c^* = f(k^*) - sf(k^*) = f(k^*) - (\delta + n)k^*.$$

The level of capital that corresponds to the maximum consumption, which we call the golden-rule level of capital, must satisfy the first-order condition:

$$f'(k^*_{gold}) = \delta + n.$$

At the golden-rule level, marginal product of capital (MPK) equals the depreciation rate plus the population growth rate.

Recall that the steady-state level of capital is an increasing function of the saving rate, $k^*(s)$. We might adjust s to achieve the golden-rule level of capital. Suppose that $k^*(s) < k^*_{gold}$. Since $\frac{dk^*}{ds} > 0$, we might increase the saving rate to achieve the golden-rule level. If the initial level of capital is higher than the golden-rule level, then we might decrease the saving rate to achieve the golden-rule level.

For example, suppose $n = 0, \delta = 0.1$, $f(k) = k^{1/2}$. Then solving for the steady-state level, we obtain

$$sk^{*1/2} = 0.1k^*.$$

We then have $k^*(s) = 100s^2$. Suppose $s = 0.3$, we obtain the steady-state level of capital in this economy.

$$k^* = 9.$$

The golden-rule level, however, is obtained from

$$\frac{1}{2}{k^*_{gold}}^{-1/2} = 0.1,$$

which gives k^*_{gold}=25. In this economy, the steady-state level of capital is too low. We might increase the saving rate to achieve the golden rule. Which saving rate corresponds to the golden rule? We solve $100s^2_{gold} = 25$ and obtain $s_{gold} = 0.5$. To see how the economy adjusts to a change in saving rate, the experiment using the Excel spreadsheet (Solow1.xlsx) is again encouraged.

To see how population growth affects steady-state income, we still examine the equation characterizing the steady state,

$$sf(k^*) = (\delta + n)k^*.$$

Using the implicit function theorem, we have

$$\frac{dk^*}{dn} = -\frac{-k^*}{sf'(k^*) - (\delta + n)} < 0.$$

Hence higher population growth leads to lower per cap capital, output, and income in steady state. Empirically, we do see negative correlation between population growth and income per capita. However, the negative correlation does not prove that higher population growth *causes* lower living standard. In fact, population growth itself is endogenous. In wealthy societies, for example, costs of raising and educating children are high. Hence people choose to have fewer children.

Solow Model II

In the first Solow model, there is no sustainable growth in income per capita. This may be true for many poor countries in the world, or the world before the industrial revolution. But there are a number of countries that have experienced sustained growth in the span of several decades or centuries (e.g., the United Kingdom and the United States).

To allow for such sustainable growth in per capita output/income, we introduce technological progress into the second Solow model. We assume that the economy can be characterized by a labor-augmenting production function, $Y_t = F(K_t, E_t L_t)$, where

- F is a constant-return-to-scale function,
- $E_t = E_0 e^{gt}$ is technology level, which grows at a constant rate g, and
- $L_t = L_0 e^{nt}$ denotes population, which grows at a constant rate n.

We also assume:

- Closed economy ($NX = 0$), no government spending ($G = 0$).
- The saving rate is constant: $C = (1 - s)Y$.
- Capital depreciates at constant rate δ.

We let $y = \frac{Y}{EL}$ and $k = \frac{K}{EL}$. y is called the output per effective worker (p.e.w.), and k is the amount of capital p.e.w. We have

$$y = \frac{Y}{EL} = \frac{F(K, EL)}{EL} = F(k, 1).$$

As in the first Solow model, we define $f(k) \equiv F(k, 1)$, and write

$$y = f(k).$$

f(k) can be called the "per effective worker production function" (p.e.w. production function), which characterizes how much output one effective worker could produce using *k* units of capital. As in the first Solow model, we assume

$$f(0) = 0,\ f'(k) > 0,\ f''(k) < 0.$$

Note that $f'(k)$ is the marginal product of capital (MPK). We also assume:

$$\lim_{k \to 0} f'(k) = \infty,\ \lim_{k \to \infty} f'(k) = 0$$

Note that $\dot{E}_t = gE_t$ and $\dot{L}_t = nL_t$. The accumulation of total capital stock follows

$$\dot{K}_t = sF(K_t, E_t L_t) - \delta K_t.$$

Then the dynamics of p.e.w. capital accumulation is then characterized by

$$\dot{k}_t \equiv \frac{d}{dt}\left(\frac{K_t}{E_t L_t}\right) = \frac{\dot{K}_t}{E_t L_t} - \frac{K_t \dot{L}_t}{E_t L_t^2} - \frac{K_t \dot{E}_t}{L_t E_t^2}$$
$$= sf(k_t) - (\delta + n + g)k_t.$$

The steady state capital p.e.w., k^*, is then characterized by the following equation,

$$sf(k^*) - (\delta + n + g)k^* = 0.$$

At steady state, the capital per effective worker is a constant,

$$\frac{K_t}{E_t L_t} = k^*.$$

This implies that the total output, $Y_t = E_t L_t f(k^*)$, grows at the constant rate $n + g$ and that the per capita output, $\frac{Y_t}{L_t} = E_t f(k^*)$, grows at the constant rate g. Thus the Solow Model II, by incorporating the factor of technological progress, is able to explain sustained growth in per capita terms.

Using the same technique as in the previous section, we may analyze the effect of saving rate on the steady-state p.e.w. capital. There is also an optimal saving rate that corresponds to the golden rule of capital p.e.w., which results in maximum consumption. We leave these analyses to exercises.

Recall that in a competitive economy, the real wage equals marginal product of labor (MPL) and the real rental price of capital equals the marginal product of capital (MPK). At the steady state, we have

$$MPL = \frac{\partial Y}{\partial L} = \frac{\partial}{\partial L}\left(ELf\left(\frac{K}{EL}\right)\right) = E\big(f(k^*) - k^* f'(k^*)\big).$$

$$MPK = \frac{\partial Y}{\partial K} = \frac{\partial}{\partial K}\left(ELf\left(\frac{K}{EL}\right)\right) = f'(k^*).$$

Hence the Solow Model II implies that the real wage grows with the technological progress and that the real return to capital remains constant. Empirical evidence largely supports this conclusion.

A Simple Endogenous Model

In the Solow model, technological progress is assumed. Endogenous models treat the technology progress as an outcome of economic activities, or in other words, as an endogenous process. In this section we introduce a simple model that does generates sustainable growth without making an exogenous assumption on the technological progress.

The model assumes that the population is constant and the technology of the economy is linear. Specifically, we assume

$$Y_t = AK_t,$$

where Y_t is output, K_t is capital stock that includes "knowledge", and A is a constant. The capital accumulation follows

$$\dot{K}_t = sY_t - \delta K_t.$$

It is obvious that

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = sA - \delta.$$

As long as $sA > \delta$, sustained growth is achieved without making an exogenous assumption on the technological progress. Obviously, the sustained growth depends on high saving and investment. This is made possible by the linear technology, which enjoys the constant return to capital. In contrast, the Solow model assumes diminishing return to capital. To make the assumption of constant return to capital more reasonable, we should understand that the capital stock in this model includes "knowledge".

Growth Accounting

A nation can achieve economic growth, either by accumulating factor inputs (e.g., labor and capital), or by increasing efficiency ("technology" of the aggregate economy). The job of growth accounting is to assess the contribution of factor inputs and efficiency gain to economic growth. Note that since the marginal product of capital is generally believed to be declining as the capital stock increases, the economic growth that relies on capital accumulation is generally considered unsustainable, and thus "bad". Obviously, the growth that relies on population growth is "bad" too, since it does not improve the living standard of average people. In contrast, if a substantial part of economic growth comes from efficiency gain, then the growth is considered sustainable, and thus "good".

We assume that the economy can be characterized by

$$Y_t = A_t F(K_t, L_t),$$

where $F(\cdot,\cdot)$ is a constant-return-to-scale production function and A_t is a positive process that measures the overall efficiency of the economy. Note that here, technological progress augments not only labor (as in Solow II), but also capital. In this sense we call A_t the *total factor productivity*.

Taking total differentiation and divide both sides by Y_t,

$$\frac{\dot{Y}_t}{Y_t} = \frac{A_t F_1 \times K_t}{Y_t} \times \frac{\dot{K}_t}{K_t} + \frac{A_t F_2 \times L_t}{Y_t} \times \frac{\dot{L}_t}{L_t} + \frac{\dot{A}_t}{A_t}.$$

Note that $A_t F_1 \equiv A_t \frac{\partial F(K_t, L_t)}{\partial K_t}$ is the marginal product of capital and $A_t F_2 \equiv A_t \frac{\partial F(K_t, L_t)}{\partial L_t}$ is the marginal product of labor. If we assume that the markets for factor inputs are competitive, then $\frac{A_t F_1 \times K_t}{Y_t}$ and $\frac{A_t F_2 \times L_t}{Y_t}$ are the income shares of capital and labor, respectively. Let $\alpha = \frac{A_t F_1 \times K_t}{Y_t}$, we have

$$\frac{\dot{Y}_t}{Y_t} = \alpha \frac{\dot{K}_t}{K_t} + (1 - \alpha) \frac{\dot{L}_t}{L_t} + \frac{\dot{A}_t}{A_t}.$$

In this equation, the growth rate of the total output $(\frac{\dot{Y}_t}{Y_t})$ is decomposed into three components, the growth of capital stock $\left(\frac{\dot{K}_t}{K_t}\right)$, the growth of labor $\left(\frac{\dot{L}_t}{L_t}\right)$, and technological progress $\left(\frac{\dot{A}_t}{A_t}\right)$. In empirical analysis, this equation can be estimated by least squares. The term $\frac{\dot{A}_t}{A_t}$ invariably appears as the residual process in empirical analysis. So the term $\frac{\dot{A}_t}{A_t}$ is called the **Solow residual**. It is the changes in output that cannot be explained by changes in factor inputs.

Case Study: Source of Growth in US.

(To be continued)