1. Model Selection, Estimation, and Diagnostics. In this exercise we use the dataset volseries.csv, which contains three time series variables: $x$ and $y$.

(1) Obtain the correlograms (ACF and PACF) of $x$, $y$, $x^2$, and $y^2$.

(2) Estimate an ARCH(1) model for $x$, which is given by

$$x_t = \sigma_t \varepsilon_t,$$

where $\varepsilon_t$ is iid(0,1) and $\sigma_t$ is the conditional standard deviation (volatility) that satisfies,

$$\sigma_t^2 = c + ax_{t-1}^2.$$

(EViews tip: Use menu “Object” → “New Object”; In the Equation Specification, choose ARCH in the menu “Method” in “Estimation settings”. In the “Mean Equation Specification”, type “x”. In the “ARCH Specification”, choose Order ARCH to be 1 and GARCH 0.)

(3) Diagnostics. To check if a ARCH/GARCH model is adequate, we should look at the estimated standardized residual,

$$\hat{\varepsilon}_t = \frac{x_t}{\hat{\sigma}_t}.$$

If our model is right, both $\varepsilon_t$ and $\varepsilon_t^2$ should be white noise. In particular, if the latter is serially correlated, then our model does not fully capture the volatility clustering in $x$. If this is the case, we should improve our model by using more lags or use GARCH.

Of course, EViews has estimated $\hat{\varepsilon}_t$ for you. You only need to check the correlograms of them.

(EViews tip: Use menu “View” → “Residual Tests”, choose “Correlogram - Q Statistic”
to test whether $\hat{\varepsilon}_t$ is white noise and choose “Correlogram Squared Residuals” to test whether $\hat{\varepsilon}_t^2$ is white noise.)

(3) You may find that the ARCH(1) is not adequate for $x$. Try estimating an ARCH(2) model for $x$ and perform diagnostics for this new model.

(4) Estimate a GARCH(1,1) model for $y$. Perform model diagnostics.

(5) Plot conditional volatility. In financial applications, the conditional volatility $\sigma_t$ in an ARCH/GARCH model usually describes “risks” perceived by investors. Of course, EViews has also estimated $\sigma_t$ for you. Use menu “View” → “Conditional SD Graph”.

(6) One-step-ahead forecast of conditional variance $\sigma_{n+1}^2$. Recall from the lecture that, for GARCH(1,1) models,

$$\hat{\sigma}_{n+1}^2 = \hat{c} + \hat{\alpha}y_n^2 + \hat{\beta}\hat{\sigma}_n^2.$$  

(EViews tip: You can obtain $\hat{\sigma}_t^2$ by using menu “Proc”→“Make GARCH Variance Series”.

(7) Make a two-step-ahead forecast of conditional variance of $y$.

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2. **Modeling Volatility Clustering of IBM Stock Return.** In this exercise we use the dataset ibm.csv, which contains daily returns on IBM stock and S&P 500 Index.

(1) Estimate the following model for $ibm$,

$$ibm_t = \beta_0 + u_t, \quad (1)$$

where

$$u_t \sim \text{GARCH}(1,1).$$

This is,

$$u_t = \sigma_t \varepsilon_t, \quad (2)$$
where $\varepsilon_t$ is iid(0,1) and $\sigma_t$ satisfies,

$$\sigma_t^2 = c + au_{t-1}^2 + b\sigma_{t-1}^2. \tag{3}$$

The reason why we add a constant term $\beta_0$ is that stocks normally yield a nonzero return in the long run. Notice that

$$\beta_0 = \mathbb{E}(ibm_t|ibm_{t-1}, ibm_{t-2}, \ldots).$$

Hence Equation (1) is usually called the “equation of conditional mean” and Equation (2) and (3) are called the “equation of conditional variance”.

If we suspect that $ibm_t$ may be serially correlated, we improve (1) by adding AR and MA terms. For example, we may write

$$ibm_t = \beta_0 + \beta_1 ibm_{t-1} + \alpha u_{t-1} + u_t, \tag{4}$$

where $u_t \sim \text{GARCH}(1,1)$. This model is typically called ARMA(1,1)-GARCH(1,1). For now, we concentrate on the model in (1), since it is well known the returns on “star” stocks such as IBM are not serially correlated.

(EViews tip: To include the constant $\beta_0$, the only thing you need to do is to type “ibm c” in the “Mean Equation Specification”. To include AR(1) and MA(1) terms, simply type “ibm c ar(1) ma(1)”.)

(2) Perform model diagnostics. Is this model adequate?

(3) Plot conditional volatility.

(4) One-step-ahead forecast of conditional volatility. Note that, in contrast to the first problem, the residual $u_t$ should be estimated. Eviews has done that for you. Use menu “Proc” → “Make Residual Series” and choose “ordinary”. If you choose “standardized”, you will get standardized residual $\hat{\varepsilon}_t$. 
(5) Estimate an ARMA(1,1)-GARCH(1,1) model for sp500, the return on the S&P 500 Index. Perform model diagnostics.

(6) Plot conditional volatility. Compare with that of the IBM stock.