Asset Pricing Theory Problem Set 4

Due: Next Lecture

1. Consider a market that consists of a money account M_t with constant riskfree rate r, a stock S_t, a European call C_t on the stock with strike price K, a European put P_t on the stock with the same strike price, and a stock forward on S_t at price K. All derivatives expire/deliver at time T. And we assume there is no arbitrage opportunity in this market.

(1) For $t \in [0, T]$, we know that the price at time t of a European call expiring at time T is

$$C_t = \tilde{\mathbb{E}}_t \left(e^{-r(T-t)} \max(S_T - K) \right),$$

where S_T is the underlying asset price at time T and K is the strike price. Then what is the price for the put at time t?

(2) Using the no arbitrage condition and the risk neutral probability, show that the forward price is

$$F_t = S_t - e^{-r(T-t)} K.$$

(3) Show the following put-call parity relationship holds,

$$C_t - P_t = F_t.$$

(4) Now, given a date $t_0 \in (0, T)$, consider a "chooser option" that gives the right at time t_0 to choose to own either the call or the put. Show that that at time t_0, the value of the chooser option is

$$C_{t_0} + \max(0, -F_{t_0}).$$

(5) Show that the value of the chooser option at time 0 is the sum of the value of a call expiring at time T with strike price K and the value of a put expiring at time t_0 with strike price $\exp(-r(T - t_0))K$.